Second induction example

- Rosen, section 3.3, question 2:
  - Show the sum of the first n positive even integers is $n^2 + n$
  - Rephrased:

$$\forall n \, \mathrm{P}(n) \text{ where } \mathrm{P}(n) = \sum_{i=1}^{n} 2i == n^2 + n$$

- The three parts:
  - Base case
  - Inductive hypothesis
  - Inductive step

Second induction example, continued

- Base case: Show P(1):

$$P(1) = \sum_{i=1}^{1} 2(i) == 1^2 + 1$$
$$= 2 == 2$$

- Inductive hypothesis: Assume

$$P(k) = \sum_{i=1}^{k} 2i == k^2 + k$$

- Inductive step: Show

$$P(k+1) = \sum_{i=1}^{k+1} 2i == (k+1)^2 + (k+1)$$

Second induction example, continued

- Recall our inductive hypothesis:

$$P(k) = \sum_{i=1}^{k} 2i == k^2 + k$$

$$\sum_{i=1}^{k+1} 2i == (k+1)^2 + k + 1$$

$$2(k+1) + \boxed{\sum_{i=1}^{k} 2i} == (k+1)^2 + k + 1$$

$$2(k+1) + \boxed{k^2 + k} == (k+1)^2 + k + 1$$

$$k^2 + 3k + 2 == k^2 + 3k + 2$$

Notes on proofs by induction

- We manipulate the *k*+1 case to make part of it look like the *k* case
- We then replace that part with the other side of the *k* case

$$\mathrm{P}(k) = \sum_{i=1}^{k} 2i == k^2 + k$$

$$\sum_{i=1}^{k+1} 2i == (k+1)^2 + k + 1$$

$$2(k+1) + \boxed{\sum_{i=1}^{k} 2i} == (k+1)^2 + k + 1$$

$$2(k+1) + \boxed{k^2 + k} == (k+1)^2 + k + 1$$

$$k^2 + 3k + 2 == k^2 + 3k + 2$$

Third induction example

- Rosen, question 7: Show

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

- Base case: n = 1

$$\sum_{i=1}^{1} i^2 = \frac{1(1+1)(2+1)}{6}$$

$$1^2 = \frac{6}{6}$$

$$1 = 1$$

- Inductive hypothesis: assume

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$

Third induction example

- Inductive step: show $\sum_{i=1}^{k+1} i^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$

$$\sum_{i=1}^{k+1} i^2 = \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6}$$

$$(k+1)^2 + \sum_{i=1}^{k} i^2 = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$(k+1)^2 + \frac{k(k+1)(2k+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6}$$

$$6(k+1)^2 + k(k+1)(2k+1) = (k+1)(k+2)(2k+3)$$

$$2k^3 + 9k^2 + 13k + 6 = 2k^3 + 9k^2 + 13k + 6$$

$$\sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6}$$