What is induction?

- A method of proof
- It does not generate answers: it only can prove them
- Three parts:
  - Base case(s): show it is true for one element
  - Inductive hypothesis: assume it is true for any given element
    - **Must be clearly labeled!!!**
  - Show that if it true for the next highest element

Induction example

- Show that the sum of the first *n* odd integers is n^2
  - Example: If $n = 5$, $1+3+5+7+9 = 25 = 5^2$
  - Formally, Show

$$\forall n \, \mathrm{P}(n) \text{ where } \mathrm{P}(n) = \sum_{i=1}^{n} 2i - 1 == n^2$$

- Base case: Show that P(1) is true

$$\mathrm{P}(1) = \sum_{i=1}^{1} 2(i) - 1 == 1^2$$

$$= 1 == 1$$

Induction example, continued

- Inductive hypothesis: assume true for *k*
  - Thus, we assume that P(*k*) is true, or that

$$\sum_{i=1}^{k} 2i - 1 == k^2$$

  - Note: we don't yet know if this is true or not!

- Inductive step: show true for *k*+1
  - We want to show that:

$$\sum_{i=1}^{k+1} 2i - 1 == (k+1)^2$$

Induction example, continued

- Recall the inductive hypothesis:

$$\sum_{i=1}^{k} 2i-1 == k^2$$

- Proof of inductive step:

$$\sum_{i=1}^{k+1} 2i-1 == (k+1)^2$$

$$2(k+1)-1+\boxed{\sum_{i=1}^{k} 2i-1} == k^2+2k+1$$

$$2(k+1)-1+\boxed{k^2} == k^2+2k+1$$

$$k^2+2k+1 == k^2+2k+1$$